Key

Math 4
Unit 2 Review

Name______________________________

Date________

1. Let $A=\begin{bmatrix}5 & 6\\-2 & 7\end{bmatrix}$ and $B=\begin{bmatrix}3 & -2\\-7 & -4\end{bmatrix}$ Without your calculator, find the following:

a. $A+2B$

$$\begin{bmatrix}5 & 6\\-2 & 7\end{bmatrix}+\begin{bmatrix}6 & -4\\-14 & -8\end{bmatrix}=\boxed{\begin{bmatrix}11 & 2\\-16 & -1\end{bmatrix}}$$

c. $B-2A$

$$\begin{bmatrix}3 & -2\\-7 & -4\end{bmatrix}-\begin{bmatrix}10 & 12\\-4 & 14\end{bmatrix}=\boxed{\begin{bmatrix}-7 & -14\\-3 & -18\end{bmatrix}}$$

b. AB

2×2

$$\begin{bmatrix}15+-42 & -10+-24\\-6+-49 & 4+-28\end{bmatrix}=\boxed{\begin{bmatrix}-27 & -34\\-55 & -24\end{bmatrix}}$$

d. BA

$$\begin{bmatrix}15+4 & 18+-14\\-35+8 & -42+-28\end{bmatrix}=\boxed{\begin{bmatrix}19 & 4\\-27 & -70\end{bmatrix}}$$

2. Solve for a and b: $\begin{bmatrix}3 & a\\5 & b\end{bmatrix}\begin{bmatrix}4\\6\end{bmatrix}=\begin{bmatrix}18\\32\end{bmatrix}$

$3\cdot4+a\cdot6=18$

$12+6a=18$

$6a=6$

$\boxed{a=1}$

$5\cdot4+b\cdot6=32$

$20+6b=32$

$6b=12$

$\boxed{b=2}$

3. Suppose that matrix A has dimensions $p\times m$ and matrix B has dimensions $m\times n$. Which of the following statements is **true**?

$A\cdot B$

$p\times m\cdot m\times n$

$B\cdot A$

$m\times n\cdot p\times m$

~~(A)~~ The product AB exists and has dimensions $m\times n$.

~~(B)~~ The product BA exists and has dimensions $p\times n$.

~~(C)~~ The product BA exists and has dimensions $m\times n$.

(D) The product AB exists and has dimensions $p\times n$.

$2 \times 3 \cdot A = 2 \times 1$

4. If $\begin{bmatrix} 5 & -2 & -1 \\ 2 & 4 & 7 \end{bmatrix} \bullet A = \begin{bmatrix} 7 \\ 76 \end{bmatrix}$, what are the dimensions of matrix A ?

$\boxed{A \text{ is } 3 \times 1.}$

5. The average number of gallons of unleaded gas sold daily at three different gas stations is given in the matrix below.

$$\begin{array}{cc} & \begin{array}{ccc} \text{Regular} & \text{Super} & \text{Premium} \end{array} \\ \begin{array}{c} Jack's \\ Karen's \\ Carl's \end{array} & \begin{bmatrix} 1800 & 3400 & 3000 \\ 1200 & 5000 & 3100 \\ 2000 & 8200 & 4300 \end{bmatrix} \end{array} \quad \begin{array}{c} R \\ S \\ P \end{array}\begin{bmatrix} 3.79 \\ 3.89 \\ 4.02 \end{bmatrix}$$

R S P

The selling prices for regular, super, and premium are \$3.79, \$3.89 , \$4.02 , respectively. Use a product of **2** matrices to find the gas revenue for each station. **Be sure to show the product of the matrices you use to arrive at your answer.**

Jack's \$32,108

Karen's \$36,460

Carl's \$56,764

6. Let $A = \begin{bmatrix} 3 & -4 & 3 \\ 2 & z & 1 \\ 5 & -6 & 4x+1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & y+5 & 3 \\ 2 & 0 & 1 \\ 5 & -6 & x-2 \end{bmatrix}$

If $A = B$, find the values of x, y and z.

$4x + 1 = x - 2$

$3x = -3$

$\boxed{x = -1}$

$-4 = y + 5$

$\boxed{-9 = y}$

$\boxed{z = 0}$

7. During the 2000 Summer Olympic games in Sydney, several U.S. athletes won multiple medals. Three athletes who won more than one medal were swimmers Dara Torres and Gary Hall, Jr. and runner Marion Jones. Dara Torres won 2 gold and 3 bronze medals. Gary Hall, Jr. won 1 gold, 1 silver, and 1 bronze medal. Marion Jones won 3 gold and 2 bronze medals.

a. Fill in the matrix below so that it indicates the medals won by each athlete.

$$\begin{array}{r} \\ \text{Gold} \\ \text{Silver} \\ \text{Bronze} \end{array} \begin{array}{c} \begin{array}{ccc} \text{DT} & \text{GH} & \text{MJ} \end{array} \\ \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 0 \\ 3 & 1 & 2 \end{bmatrix} \end{array}$$

b. The U.S. Olympic Committee gave cash bonuses to any U.S. athlete winning a medal. They gave \$15,000 for gold, \$10,000 for silver, and \$7,500 for bronze. Put the cash bonuses into a 1×3 matrix. Be sure to label your matrix properly.

$$\begin{bmatrix} 15000 & 10000 & 7500 \end{bmatrix}$$

c. Use matrix multiplication to find the amount of bonus money awarded to each athlete. Show the order in which you multiplied the matrices and properly label the product matrix.

$$\begin{bmatrix} 15000 & 10000 & 7500 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 0 \\ 3 & 1 & 2 \end{bmatrix} = \boxed{\begin{array}{ccc} \text{DT} & \text{GH} & \text{MJ} \\ \$52,500 & 32,500 & 60,000 \end{array}}$$

8. Let $B = \begin{bmatrix} 1 & 0 & -6 \\ -2 & 5 & 4 \end{bmatrix}$. What is the additive inverse of matrix B?

a. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

b. $\begin{bmatrix} -1 & 0 & 6 \\ 2 & -5 & -4 \end{bmatrix}$

c. $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

e. there is no additive inverse of matrix B.

$ad - bc$

9. Without using a calculator, find the following: $\begin{array}{c} a \quad b \\ \begin{bmatrix} 9 & 7 \\ -3 & 5 \end{bmatrix} \\ c \quad d \end{array} \cdot \begin{bmatrix} 9 & 7 \\ -3 & 5 \end{bmatrix}^{-1} =$

det is $\neq 0$, so it is invertible.

$$\boxed{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$$

In numbers 10 - 12, solve the following systems using matrices. Make sure to write your matrix equation as well as the solution matrix.

10. $\begin{cases} 7x+4y=7.6 \\ 6x+5y=19.4 \end{cases}$

$$\begin{bmatrix} 7 & 4 \\ 6 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7.6 \\ 19.4 \end{bmatrix}$$

$$\begin{bmatrix} 7 & 4 \\ 6 & 5 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 7.6 \\ 19.4 \end{bmatrix} = \boxed{\begin{bmatrix} -3.6 \\ 8.2 \end{bmatrix} \quad \begin{matrix} x=-3.6 \\ y=8.2 \end{matrix}}$$

11. $\begin{cases} 8x-3y=6 \\ 4y-5x=9 \end{cases}$ $\longrightarrow$ * x & y terms switched!!

$$*\begin{bmatrix} 8 & -3 \\ -5 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} 8 & -3 \\ -5 & 4 \end{bmatrix}^{-1}\begin{bmatrix} 6 \\ 9 \end{bmatrix} = \boxed{\begin{bmatrix} 3 \\ 6 \end{bmatrix}}$$

$x=3$

$y=6$

12. $\begin{cases} x+z=6 \\ -3y+z=7 \\ 2x+y+3z=15 \end{cases}$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 7 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & 1 \\ 2 & 1 & 3 \end{bmatrix}^{-1}\begin{bmatrix} 6 \\ 7 \\ 15 \end{bmatrix} = \boxed{\begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}}$$

$x=2$

$y=-1$

$z=4$